# Adaptive Backstepping Robust Control of Nonlinear Spray Boom System

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***Abstract*—In this paper, a two-asymmetric hydraulic cylinder electro-hydraulic servo system model is established for the spray boom. Because of spray boom vibration and tilt ,which are caused by boom sprayer during its walking, and hydraulic cylinder parameter drift due to environmental changes, an adaptive backstepping control algorithm with an integral-type Lyapunov function is designed to ensure the balance of the boom and a certain height with the ground. Moreover，a higher performance nature of the proposed nonlinear adaptive robust control approach was presented in comparison to the proposed control without adaptation laws. A simulation results showed that the nonlinear algorithm has an excellent performance for the specified tracking task and strong robustness.**

***Index Terms*—spray boom, electro-hydraulic servo system, adaptive backstepping, uncertain parameters**

## I. Introduction

The sprayer is an ideal field crop plant protection machine with a wide spray range, uniform spraying and high working efficiency. Because of the external disturbance caused by the undulation of road surface, the sprayer working in the field produces uneven spraying and scratching crops. In severe cases, the end of the boom may touch the ground. Therefore, the positional balance control of the spray boom is an very important issue, which provides a theoretical foundation for reasonable matching of structural parameters and active control. It is of great significance to protect the spray boom and improve the spray effect [1]-[4].

For the vibration problem of the spray boom of the sprayer, domestic scholars mainly used the ultrasonic sensor to detect the actual height from ground to boom in real time. Then the system sends out control signals of hydraulic cylinder to adjust spray boom height and balance. An online control system of spray boom height and balance was designed in [5], [6]. A electro-hydraulic servo system used in the boom system has the advantages of fast response speed, low cost and large carrying capacity, and was widely used in the spray boom damping system. However, the electro-hydraulic servo system has severe nonlinearity, which brought great difficulties to the design of the controller when nonlinear uncertain parameters disturbance occurs. In the past, linear control theory has been used in much of the work on hydraulic control systems, such as in [7]-[9]. some important dynamic information may be lost if the electro-hydraulic servo system is linearized. Thus, it is significant to design a nonlinear control method suitable for hydraulic system. The backstepping control algorithm is structured and standardized, and can be used to control nonlinear systems with triangular structure. A nonlinear adaptive robust control algorithm is presented for a single-rod electro-hydraulic actuator with unknown nonlinear parameters in [10]-[13]. The nonlinear adaptive control law has better performance than conventional linear controllers.

In this paper, based on the boom balance control system of the [4], two asymmetric hydraulic cylinders electro-hydraulic servo system is established. An improved adaptive backstepping control method is proposed for the electro-hydraulic servo system with uncertain parameters. For the nonlinear uncertain parameters, the state variable conversion method of the [11] is used. By introducing a linear transformation of a set of state variables, it is appropriately transformed into a state space model that can be processed by the backstepping algorithm. Then, the construction idea of Lyapunov function and the design of adaptive law in [12] are used to overcome the affect of drift of uncertain parameters. A simulation results show that the control method and the adaptive scheme can obtain excellent performance when tracking the position signal trajectory, even if there are uncertain nonlinear parameters due to changes in environmental conditions.

## II. Modeling of Spray Boom System

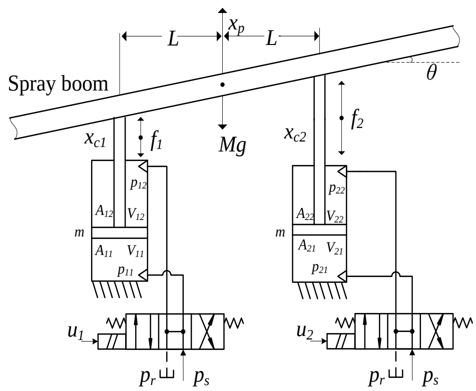


Figure 1. Schematic diagram of the spray boom system

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The spray boom system shown in Fig. 1. The goal of this paper is that the height of the boom is maintained and the vibration of the boom is suppressed. Therefore two variables, the vertical displacement $x_p$ and the tilt angle $\theta$, track any specified motion trajectory as closely as possible.

The vertical displacement of the boom is related to the displacement of the pistons of the two hydraulic cylinders as follows:

$$x_p = \frac{\left(x_{c1} + x_{c2}\right)}{2} \tag{1}$$

where $x_{c1}$ and $x_{c2}$ are the piston displacements of the two hydraulic cylinders, respectively.

The inclination angle of the boom is related to the displacement of the pistons of the two hydraulic cylinders as follows:

$$\theta = \arctan(\frac{x_{c2} - x_{c1}}{2L}) \tag{2}$$

Two dynamic equations can be obtained by applying Newton's second law and law of rotation to the boom system:

$$f_1 + f_2 - Mg = M\ddot{x}_p \tag{3}$$

$$(f_2 - f_1)L = J\ddot{\theta} \tag{4}$$

where $f_1$ and $f_1$ are the interaction forces between the two hydraulic cylinders and the spray boom, respectively, $M$ is the mass of the spray boom, $L$ is the distance from the contact point of the hydraulic cylinder to the spray boom to the center of gravity of the spray boom, $J$ is rotational inertia of spray boom.

The dynamics of the force balance for two asymmetric hydraulic cylinders can be described by

$$p_{i1}A_{i1} - p_{i2}A_{i2} - B_i x_{ci} - f_i - mg = m\ddot{x}_{ci} \quad \left(i = 1, 2\right) \tag{5}$$

where $p_{i1}$ and $p_{i2}$ are the pressure inside the two chambers of the ith cylinder, respectively, $A_{i1}$ and $A_{i2}$ care the ram area of the two chambers, respectively, $B_i$ is the viscous damping coefficient, $f_i$ is the interaction force between the ith hydraulic cylinder and the boom, $m$ is the piston mass (assuming that the pistons of two hydraulic cylinders have the same mass), $x_{ci}$ is the stroke of the ith hydraulic cylinder.

The effective working area of the pressure in the two chambers of the cylinder is not equal. The flow gain in the hydraulic cylinder changes as the piston moves back and forth. The throttle equation of the hydraulic cylinder can be expressed by

$$Q_{i1} = g_{11}R_{i1}u_i \tag{6}$$

$$Q_{i2} = g_{12}R_{i2}u_i \tag{7}$$

where $Q_{i1}$ is the rodless chamber inlet flow rate of the ith hydraulic cylinder, $Q_{i2}$ is the rod chamber return flow of the ith hydraulic cylinder, $g_i = k_{qi}k_{vi}$ is voltage-flow gain, $k_{qi1} = C_{di}\omega_{i1}\sqrt{2/\rho}$, $k_{qi2} = C_{di}\omega_{i2}\sqrt{2/\rho}$ are the spool displacement-flow gain coefficients of the ith servo valve, $K_{vi}$ is the spool voltage-displacement gain, $C_{di}$ is the discharge coefficient of the ith servo valve, $\omega_{i1}$ and $\omega_{i2}$ are area gradients, $\rho$ is the oil density, $u_i$ is the servo valve input voltage signal,

$$R_{i1} = s\left(u_i\right)\sqrt{p_s - p_{i1}} + s\left(-u_i\right)\sqrt{p_{i1}}$$

$$R_{i1} = s\left(u_i\right)\sqrt{p_s - p_{i1}} + s\left(-u_i\right)\sqrt{p_{i1}}$$

where $p_s$ is the supply pressure, define function

$$s\left(u_i\right) = \begin{cases} 1 & u_i \geq 0 \\ 0 & u_i < 0 \end{cases}$$

Each hydraulic cylinder is controlled by a servo valve, and the external leakage of the hydraulic cylinder is small and negligible. The pressure dynamics of each hydraulic cylinder can be expressed by

$$\frac{V_{i1} + A_{i1}x_{ci}}{\beta_e}\dot{p}_{i1} = Q_{i1} - A_{i1}\dot{x}_{ci} - C_t\left(p_{i1} - p_{i2}\right) \tag{8}$$

$$\frac{V_{i2} - A_{i2}x_{ci}}{\beta_e}\dot{p}_{i2} = -Q_{i2} + A_{i2}\dot{x}_{ci} + C_t\left(p_{i1} - p_{i2}\right) \tag{9}$$

where $V_{i1}$ and $V_{i2}$ are the original total volumes of the two chambers of the hydraulic cylinder, respectively (including the volume of the servo valve, pipelines, and cylinder chambers), $\beta_e$ is the effective bulk modulus of the hydraulic fluid, $C_t$ is the internal leakage coefficient of the hydraulic cylinder.

Define the state variables as $x_{i1} = x_{ci}$, $x_{i2} = \dot{x}_{ci}$, $x_{i3} = p_{i1}$, $x_{i4} = p_{i2}$ mean two hydraulic cylinders. The entire system, including (3)-(9) can be expressed in a state- space form as follows:

$$\dot{x}_{11} = x_{12}$$

$$\dot{x}_{21} = x_{22}$$

$$M\begin{bmatrix}\dot{x}_{12} \\ \dot{x}_{22}\end{bmatrix} = \begin{bmatrix}A_{11}x_{13} - A_{12}x_{14} \\ A_{21}x_{23} - A_{22}x_{24}\end{bmatrix} - G - B\begin{bmatrix}x_{12} \\ x_{22}\end{bmatrix}$$

$$\dot{x}_{13} = \frac{\beta_e}{V_{11} + A_{11}x_{11}}\left(-A_{11}x_{12} - C_t\left(x_{13} - x_{14}\right) + g_{11}R_{11}u_1\right) \quad (10)$$

$$\dot{x}_{14} = \frac{\beta_e}{V_{12} - A_{12}x_{11}}\left(A_{12}x_{12} + C_t\left(x_{13} - x_{14}\right) - g_{12}R_{12}u_1\right) \quad (11)$$

$$\dot{x}_{23} = \frac{\beta_e}{V_{21} + A_{21}x_{21}}\left(-A_{21}x_{22} - C_t\left(x_{23} - x_{24}\right) + g_{21}R_{21}u_2\right) \quad (12)$$

$$\dot{x}_{24} = \frac{\beta_e}{V_{22} - A_{22}x_{21}}\left(A_{22}x_{22} + C_t\left(x_{23} - x_{24}\right) - g_{22}R_{22}u_2\right) \quad (13)$$

where $M = \begin{bmatrix} m + \frac{ML^2 + J}{4L^2} & \frac{ML^2 - J}{4L^2} \\ \frac{ML^2 - J}{4L^2} & m + \frac{ML^2 + J}{4L^2} \end{bmatrix}$,

$G = \begin{bmatrix} \frac{Mg}{2} + mg \\ \frac{Mg}{2} + mg \end{bmatrix}$, $B = \begin{bmatrix} B_1 & 0 \\ 0 & B_2 \end{bmatrix}$

Matrix $M$ couples the velocity state quantities together, introducing a new set of state variables for later use of the control algorithm [11].

To simplify the state-space equation, define parameters set $\zeta = \left[\zeta_{i1}\zeta_{i2}\zeta_{i3}\zeta_{i4}\zeta_{i5}\zeta_{i6}\right]^T$ as $\zeta_{11} = b_1$, $\zeta_{21} = b_2$, $\zeta_{i2} = A_{i1}\beta_e$, $\zeta_{i3} = \beta_e C_t$, $\zeta_{i4} = \beta_e g_{i1}$, $\zeta_{i5} = \beta_e g_{i2}$, $\zeta_{i6} = \beta_e A_{i2}$; parameters set $\beta = \left[\beta_{i1}\beta_{i2}\right]^T$ as $\beta_{i1} = V_{i1}/A_{i1}$, $\beta_{i2} = V_{i2}/A_{i2}$; parameters set $\varepsilon_a = \left[\varepsilon_{i1}, \varepsilon_{i2}\right]^T$ as $\varepsilon_{i1} = \zeta_{i5}\beta_{i1}$, $\varepsilon_{i2} = \zeta_{i4}\beta_{i2}$; parameters set $\varepsilon_b = \left[\varepsilon_{i3}, \varepsilon_{i4}, \varepsilon_{i5}, \varepsilon_{i6}, \varepsilon_{i7}, \varepsilon_{i8}, \varepsilon_{i9}\right]^T$ as $\varepsilon_{i5} = \zeta_{i2} - \zeta_{i6}$, $\varepsilon_{i3} = \beta_{i2}\zeta_{i2} + \beta_{i1}\zeta_{i6}$, $\varepsilon_{i4} = \left(\beta_{i1} + \beta_{i2}\right)\zeta_{i3}$, $\varepsilon_{i6} = \phi_{i1}\zeta_{11}$, $\varepsilon_{i7} = \phi_{i2}\zeta_{11}$, $\varepsilon_{i8} = \phi_{i1}\zeta_{21}$, $\varepsilon_{i9} = \phi_{i2}\zeta_{21}$; parameters set $\phi = \left[\phi_{11}, \phi_{12}\right]^T$ as $\phi_{i1} = \beta_{i1}\beta_{i2}$, $\phi_{i2} = \beta_{i2} - \beta_{i1}$. Simultaneous (11)-(13) and defined parameters, state variables (10) is transformed as

$$\dot{\bar{x}}_{11} = \bar{x}_{12}$$

$$\bar{M}\begin{bmatrix} \dot{\bar{x}}_{12} \\ \dot{\bar{x}}_{22} \end{bmatrix} = P\begin{bmatrix} \bar{x}_{13} \\ \bar{x}_{23} \end{bmatrix} - PG - Y\gamma \quad (14)$$

$$\dot{\bar{x}}_{13} = \frac{\begin{bmatrix} -\varepsilon_{13}x_{12} - \varepsilon_{14}\left(x_{13} - x_{14}\right) + \varepsilon_{15}x_{11}x_{12} \\ +\left(\varepsilon_{11}R_{12} + \varepsilon_{12}R_{11} + \left(\zeta_{14}R_{12} - \zeta_{13}R_{11}\right)x_{11}\right)u_1 \end{bmatrix}}{\phi_{11} + \phi_{12}x_{11} - x_{11}^2}$$

$$\dot{\bar{x}}_{23} = \frac{\begin{bmatrix} -\varepsilon_{23}x_{22} - \varepsilon_{24}\left(x_{23} - x_{24}\right) + \varepsilon_{25}x_{21}x_{22} \\ +\left(\varepsilon_{21}R_{22} + \varepsilon_{22}R_{21} + \left(\zeta_{24}R_{22} - \zeta_{23}R_{21}\right)x_{21}\right)u_2 \end{bmatrix}}{\phi_{21} + \phi_{22}x_{21} - x_{21}^2}$$

where $\bar{M} = PMP^{-1}$, $b_1 = \left(B_1 + B_2\right)/2$, $\gamma = \left[b_1, b_2\right]^T = \left[\zeta_{11}, \zeta_{21}\right]^T$, $Y = \begin{bmatrix} \bar{x}_{12} & \bar{x}_{22} \\ \bar{x}_{22} & \bar{x}_{12} \end{bmatrix}$

## III. Assumption and Lemma

In order to facilitate the derivation of the control algorithm and meet the constraints of the actual physical model, the following assumptions are made:

Assumption 1: The desired target position $x_d$, as well as the velocity $\dot{x}_d$, the acceleration $\ddot{x}_d$ are present and bounded.

Assumption 2: $\beta_{i1}$, $\beta_{i2}$ is bounded and satisfies the following relationship:

$$L_1 < \beta_{i1} \le D,\ L_2 < \beta_{i2} \le D \quad (i = 1, 2)$$

where $L_1$ is the possible maximal value of the absolute value of load negative displacement, $L_2$ is the possible maximal positive displacement of load, $D$ is a positive integer.

Assumption 3: $\phi_{i1} + \phi_{i2}x_{i1} - x_{i1}^2 > 0, i = 1, 2$.

Assumption 4: The uncertain parameters in the system are bounded, that is, there exists a set that satisfies

$$\theta_i \in \Omega_{\theta_i} \triangleq \left[\theta_{i\min}, \theta_{i\max}\right]$$

Lemma 1: Choose a sufficiently smooth projection operator $\mathrm{Pr}oj_\sigma\{*\}$

$$\mathrm{Pr}oj_\sigma\{*\} = \begin{cases} 0, & \text{if}\ \hat{\sigma} = \sigma_{\max}\ \text{and} * > 0 \\ 0, & \text{if}\ \hat{\sigma} = \sigma_{\min}\ \text{and} * < 0 \\ *, & \text{otherwise} \end{cases}$$

It can be obtained

$$\left(\sigma - \hat{\sigma}\right)^T\left[* - \dot{\hat{\sigma}}\right] \le 0$$

Proof: If $\hat{\sigma} = \sigma_{\max}$ and $* > 0$, then $\dot{\hat{\sigma}} = 0$ and $\tilde{\sigma} = \sigma - \hat{\sigma} = \sigma - \sigma_{\max} \le 0$, so $\left(\sigma - \hat{\sigma}\right)^T\left[* - \dot{\hat{\sigma}}\right] \le 0$;

If $\hat{\sigma} = \sigma_{\min}$ and $* < 0$, then $\dot{\hat{\sigma}} = 0$ and $\tilde{\sigma} = \sigma - \hat{\sigma} = \sigma - \sigma_{\min} \ge 0$, so $\left(\sigma - \hat{\sigma}\right)^T\left[* - \dot{\hat{\sigma}}\right] \le 0$.

## IV. Design of Controller

### A. Design of Controller

In this paper, an adaptive backstepping control algorithm is designed for the state space equation (14). Define the control deviation of the system:

$$z_1 = \overline{x}_1 - x_{1d}\,,\ z_2 = \overline{x}_2 - x_{2d}\,,\ z_3 = \overline{x}_3 - x_{3d} \tag{15}$$

where $x_{1d}$ is the control target of the system, $x_{2d}$, $x_{3d}$ are virtual control inputs for $\overline{x}_2$, $\overline{x}_3$, respectively, The relationship between $x_{1d}$ and the vertical displacement and tilt angle of the boom is $x_{1d} = \left[x_{1d1}, x_{1d2}\right]^T = \left[2x_p, 2L\tan\theta\right]^T$.

Define the parameter estimation deviation of the system as

$$\tilde{\zeta} = \zeta - \hat{\zeta}\,,\ \tilde{\gamma} = \gamma - \hat{\gamma}\,,\ \tilde{\varepsilon}_a = \varepsilon_a - \hat{\varepsilon}_a\,,\ \tilde{\varepsilon}_b = \varepsilon_b - \hat{\varepsilon}_b\,,\ \tilde{\phi} = \phi - \hat{\phi}$$

where $\hat{\zeta}$, $\hat{\varepsilon}_a$, $\hat{\varepsilon}_b$, $\hat{\phi}$ are estimates of $\zeta$, $\varepsilon_a$, $\varepsilon_b$, $\phi$, respectively.

Step 1) Design the subsystem z and derive its derivatives:

$$\dot{z}_1 = \dot{\overline{x}}_1 - \dot{x}_{1d} = \overline{x}_2 - \dot{x}_{1d} \tag{16}$$

Chose virtual control input $x_{2d} = \dot{x}_{1d} - k_1 z_1$ and, combining (15) and (16), Equation (17) can be obtained

$$\dot{z}_1 = -k_1 z_1 + z_2 \tag{17}$$

If $z_2$ is small enough or tends to zero, the system tracking error is small enough that the latter tends to zero. Thus the next step is to make $z_2$ as small as possible.

Step 2) In order to make $z_2$ as small as possible, the subsystem $z_2$ is designed. Lyapunov function is selected as

$$V_2 = \frac{1}{2} z_1^T z_1 + \frac{1}{2} z_2^T \overline{M} z_2 + \frac{1}{2}\tilde{\gamma}^T \Gamma_1^{-1} \tilde{\gamma} \tag{18}$$

As follows, the time derivatives of equation (18) is obtained as

$$\begin{aligned}\dot{V}_2 &= z_2^T P z_3 + z_2^T\left(P x_{3d} - Y\dot{\hat{\gamma}} - PG - \overline{M}\dot{x}_{2d} + z_1\right)\\ &\quad - \tilde{\gamma}^T\Gamma_1^{-1}\left(\Gamma_1 Y^T z_2 + \dot{\hat{\gamma}}\right) - k_1 z_1^T z_1\end{aligned} \tag{19}$$

Combining (15) and (19), Equation (20) and virtual control input $x_{3d} = P^{-1}\left(Y\hat{\gamma} + PG + \overline{M}\dot{x}_{2d} - z_1 - k_2 z_2\right)$ can be obtained

$$\dot{V}_2 = z_2^T P z_3 - z_1^T k_1 z_1 - z_2^T k_2 z_2 + \tilde{\gamma}^T\Gamma_1^{-1}\left(-\Gamma_1 Y^T z_2 - \dot{\hat{\gamma}}\right) \tag{20}$$

If $z_3$ is small enough or tends to zero and $\tilde{\gamma}^T\Gamma_1^{-1}\left(-\Gamma_1 Y^T z_2 - \dot{\hat{\gamma}}\right) \le 0$, then $\dot{V}_2 \le 0$, so the subsystem is globally stable. The next step is to make $z_3$ as small as possible and $\tilde{\gamma}^T\Gamma_1^{-1}\left(-\Gamma_1 Y^T z_2 - \dot{\hat{\gamma}}\right) \le 0$.

Step 3) The subsystem $z_3$ is designed. Lyapunov function is selected as

$$\begin{aligned}V_3 &= \frac{1}{2} z_3^T \begin{bmatrix} \phi_{11} + \phi_{12}x_{11} - x_{11}^2 & 0 \\ 0 & \phi_{21} + \phi_{22}x_{21} - x_{21}^2 \end{bmatrix} z_3 \\ &\quad + \frac{1}{2}\tilde{\zeta}^T\Gamma_1^{-1}\tilde{\zeta} + \frac{1}{2}\tilde{\varepsilon}_a^T\Gamma_2^{-1}\tilde{\varepsilon}_a + \frac{1}{2}\tilde{\varepsilon}_b^T\Gamma_3^{-1}\tilde{\varepsilon}_b + \frac{1}{2}\tilde{\phi}^T\Gamma_4^{-1}\tilde{\phi}\end{aligned} \tag{21}$$

As follows, the time derivatives of equation (21) is obtained as

$$\dot{V}_3 = z_3^T\left\{\begin{aligned}&\frac{1}{2}\begin{bmatrix}\phi_{12}x_{12} & 0\\ 0 & \phi_{22}x_{22}\end{bmatrix} z_3 - \begin{bmatrix} x_{11}x_{12} & 0 \\ 0 & x_{21}x_{22}\end{bmatrix} z_3 \\ &+\begin{bmatrix} \varepsilon_{11}R_{12} + \varepsilon_{12}R_{11} + \left(\zeta_{15}R_{12} - \zeta_{14}R_{11}\right)x_{11} & 0 \\ 0 & \varepsilon_{21}R_{22} + \varepsilon_{22}R_{21} + \left(\zeta_{25}R_{22} - \zeta_{24}R_{21}\right)x_{21}\end{bmatrix}\begin{bmatrix}u_1\\u_2\end{bmatrix} \\ &+\begin{bmatrix} -\varepsilon_{13}x_{12} - \varepsilon_{14}\left(x_{13} - x_{14}\right) + \varepsilon_{15}x_{11}x_{12} \\ -\varepsilon_{23}x_{22} - \varepsilon_{24}\left(x_{23} - x_{24}\right) + \varepsilon_{25}x_{21}x_{22}\end{bmatrix} \\ &-\begin{bmatrix}\phi_{11} + \phi_{12}x_{11} - x_{11}^2 & 0 \\ 0 & \phi_{21} + \phi_{22}x_{21} - x_{21}^2\end{bmatrix}\dot{x}_{3d}\end{aligned}\right\} \tag{22}$$
$$-\tilde{\zeta}^T\Gamma_1^{-1}\dot{\hat{\zeta}} - \tilde{\varepsilon}_a^T\Gamma_2^{-1}\dot{\hat{\varepsilon}}_a - \tilde{\varepsilon}_b^T\Gamma_3^{-1}\dot{\hat{\varepsilon}}_b - \tilde{\phi}\Gamma_4^{-1}\dot{\hat{\phi}}$$

The controller is designed as

$$u_1 = \frac{u_{11} + u_{12} + u_{13}}{\hat{\varepsilon}_{11}R_{12} + \hat{\varepsilon}_{12}R_{11} + \left(\hat{\zeta}_{15}R_{12} - \hat{\zeta}_{14}R_{11}\right)x_{11}} \tag{23}$$

$$u_2 = \frac{u_{21} + u_{22} + u_{23}}{\hat{\varepsilon}_{21}R_{22} + \hat{\varepsilon}_{22}R_{21} + \left(\hat{\zeta}_{25}R_{22} - \hat{\zeta}_{24}R_{21}\right)x_{21}} \tag{24}$$

where,

$$u_{11} = x_{11}x_{12}z_{31} - x_{11}^2\dot{x}_{3d11} - x_{11}^2\alpha_{11}$$

$$\begin{aligned}u_{12} &= \hat{\varepsilon}_{13}x_{12} + \hat{\varepsilon}_{14}\left(x_{13} - x_{14}\right) - \hat{\varepsilon}_{15}x_{11}x_{12} + \hat{\phi}_{11}\left(\dot{x}_{3d11} + \alpha_{11}\right)\\ &-\hat{\phi}_{12}\left(-x_{11}\dot{x}_{3d11} - x_{11}\alpha_{22} + \frac{1}{2}x_{12}z_{31}\right) + \alpha_{21}\hat{\varepsilon}_{16} + \alpha_{21}x_{11}\hat{\varepsilon}_{17} + \alpha_{22}\hat{\varepsilon}_{18}\\ &+\alpha_{22}x_{11}\hat{\varepsilon}_{19} - \alpha_{21}x_{11}^2\hat{\zeta}_{11} - \alpha_{22}x_{11}^2\hat{\zeta}_{21}\end{aligned}$$

$$u_{13} = -k_3 z_{31} - \left(z_{21} + z_{22}\right)$$

$$u_{21} = x_{21}x_{22}z_{32} - x_{21}^2\dot{x}_{3d12} - x_{12}^2\alpha_{12}$$

$$\begin{aligned}u_{22} &= \hat{\varepsilon}_{23}x_{22} + \hat{\varepsilon}_{24}\left(x_{23} - x_{24}\right) - \hat{\varepsilon}_{25}x_{21}x_{22} + \hat{\phi}_{21}\left(\dot{x}_{3d12} + \alpha_{12}\right)\\ &-\hat{\phi}_{22}\left(-x_{21}\dot{x}_{3d12} - x_{21}\alpha_{24} + \frac{1}{2}x_{22}z_{32}\right) + \alpha_{23}\hat{\varepsilon}_{26} + \alpha_{23}x_{21}\hat{\varepsilon}_{27} + \alpha_{24}\hat{\varepsilon}_{28}\\ &+\alpha_{24}x_{21}\hat{\varepsilon}_{29} - \alpha_{23}x_{21}^2\hat{\zeta}_{11} - \alpha_{24}x_{21}^2\hat{\zeta}_{21}\end{aligned}$$

$$u_{23} = -k_3 z_{32} - \left(z_{21} - z_{22}\right)$$

The adaptative law is given by

$$\begin{aligned}\dot{\hat{\zeta}} = P_{roj\hat{\theta}}\{[&\Gamma_{11}\left(-\overline{x}_{12}z_{21} - \overline{x}_{22}z_{22} + \alpha_{21}x_{11}^2 z_{31} + \alpha_{23}x_{21}^2 z_{32}\right),\\ &\Gamma_{12}\left(-\overline{x}_{22}z_{21} - \overline{x}_{12}z_{22} + \alpha_{22}x_{11}^2 z_{31} + \alpha_{24}x_{21}^2 z_{32}\right),\\ &0,0,0,0,-\Gamma_{13}R_{11}x_{11}u_1 z_{31}, -\Gamma_{14}R_{21}x_{21}u_2 z_{32},\\ &\Gamma_{15}R_{12}x_{11}u_1 z_{31}, \Gamma_{16}R_{22}x_{21}u_2 z_{32}, 0, 0]\}\end{aligned} \tag{25}$$

$$\dot{\hat{\varepsilon}}_a = \left[\Gamma_{21}R_{21}u_1z_{31}, \Gamma_{22}R_{22}u_2z_{32}, \Gamma_{23}R_{11}u_1z_{31}, \Gamma_{24}R_{21}u_2z_{32}\right] \quad (26)$$

$$\dot{\hat{\varepsilon}}_b = \begin{bmatrix} -\Gamma_{31}x_{12}z_{31}, -\Gamma_{32}x_{22}z_{32}, -\Gamma_{33}\left(x_{13}-x_{14}\right)z_{31}, \\ -\Gamma_{34}\left(x_{23}-x_{24}\right)z_{32}, \Gamma_{35}x_{11}x_{12}z_{31}, \Gamma_{36}x_{21}x_{22}z_{32}, \\ -\Gamma_{37}\alpha_{21}z_{31}, -\Gamma_{38}\alpha_{23}z_{32}, -\Gamma_{39}\alpha_{21}x_{11}z_{31}, -\Gamma_{310}\alpha_{23}x_{21}z_{32}, \\ -\Gamma_{311}\alpha_{22}z_{31}, -\Gamma_{312}\alpha_{24}z_{32}, -\Gamma_{313}\alpha_{22}x_{11}z_{31}, -\Gamma_{314}\alpha_{24}x_{21}z_{32} \end{bmatrix} \quad (27)$$

$$\begin{aligned} \dot{\hat{\phi}} = [&-\Gamma_{41}\left(\dot{x}_{3d11}+\alpha_{11}\right)z_{31}, -\Gamma_{42}\left(\dot{x}_{3d12}+\alpha_{12}\right)z_{32}, \\ &\Gamma_{43}(-x_{11}\dot{x}_{3d11}-x_{11}\alpha_{22}+x_{12}z_{31}/2)z_{31}, \\ &\Gamma_{44}(-x_{21}\dot{x}_{3d12}-x_{21}\alpha_{24}+x_{22}z_{32}/2)z_{32}] \end{aligned} \quad (28)$$

Substituting cotroller (23), (24), and adaptation law (25)-(27) into equation (33), we can get

$$\dot{V}_3 \le -k_3 z_3^T z_3 - z_3^T P z_2 + \tilde{\gamma}^T P_{roj_{\hat{\theta}}}\left(Y^T z_2\right) \quad (29)$$

*B. Stability Analysis*

Theorem 1: For the state-space equation (14), under the condition that the assumptions 1-4 and Lemma 1 are satisfied, the control law (23) (24) and the adaptation law (25)-(28) are used:

1) The control deviation $z=\left[z_1, z_2, z_3\right]^T$ and the parameter estimate $\hat{\varepsilon}_a$ , $\hat{\zeta}$ , $\hat{\phi}$ , $\hat{\varepsilon}_b$ of the system are continuous and bounded.

2) The closed loop control system is stable. When $t \to \infty$, the system tracking error $\lim\limits_{t\to\infty}\left|z_1\right| \to 0$.

Proof: For the state space equation (14), select the global Lyapunov function as

$$V = V_2 + V_3 \quad (30)$$

As follows, the time derivatives of equation (30) is obtained as

$$\dot{V} = \dot{V}_2 + \dot{V}_3 \quad (31)$$

Combining equations (20), (29), (31), the time derivatives of equation (30) can be got

$$\dot{V} \le -k_1 z_1^T z_1 - k_2 z_2^T z_2 - k_3 z_3^T z_3 < 0$$

It can be known from the above equation that when $t > 0$, the system tracking error $z_1 > 0$. The closed loop control system is asymptotic stability.

## V. Experiments

The data of the simulation reference [11] and [12] as follows: load quality $M = 100kg$, force arm $L = 2m$, rotational inertia $J = 200kg \cdot m^2$, Piston mass $m = 2kg$, voltage flow gain $g_{ii} = 3.5\times10^{-8} m^3 s^{-1} V^{-1} Pa^{-1/2}$, gupply pressure $p_s = 6MPa$, tank pressure $p_r = 0$, viscous damping coefficient $\beta_e = 2000MPa$, area of rodless chamber $A_{11} = 1.2566\times10^{-3}\, m^2$, $A_{21} = 1.3854\times10^{-3} m^2$ area of rod $A_{12} = 8.765\times10^{-4} m^2$, $A_{22} = 9.6997\times10^{-4} m^2$, initial total volume of rod chamber $V_{11} = 3.2\times10^{-4} m^3$, $V_{21} = 5.0\times10^{-4} m^3$, initial total volume of rod chamber $V_{12} = 3.2\times10^{-4} m^3$, $V_{22} = 3.5\times10^{-4} m^3$; viscous damping coefficient $B_1 = 300N\cdot s\cdot m^{-1}$, $B_2 = 290N\cdot s\cdot m^{-1}$.

The control law feedback gains are designed as $k_1 = \left[210, 210\right]$, $k_2 = \left[240, 240\right]$ and $k_3 = \left[85, 85\right]$, respectively. The adaptive parameters gained are designed as

$$\Gamma_1 = 0.75\times diag\left\{10^2, 1, 0, 0, 0, 0, 10^{-6}, 10^{-6}, 10^{-6}, 10^{-6}, 0, 0\right\}$$

$$\Gamma_2 = 0.75\times diag\left\{10^{-8}, 10^{-8}, 10^{-8}, 10^{-8}\right\}$$

$$\Gamma_3 = 0.75\times diag\begin{Bmatrix} 10^{-1}, 10^{-1}, 10^{-18}, 10^{-18}, 1, 1, 10^{-2}, \\ 10^{-2}, 1, 1, 10^{-5}, 10^{-5}, 10^{-1}, 10^{-1} \end{Bmatrix}$$

$$\Gamma_4 = 0.75\times diag\left\{10^{-7}, 10^{-7}, 10^{-5}, 10^{-5}\right\}$$

When the sprayer passes through the uneven road surface, in order to keep the vertical distance between the spray boom and the ground unchanged, the desired vertical displacement is selected to be 0.5 m, and the tracking error curve is shown in Fig. 2. The settling time of the adaptive backstepping control algorithm for nonlinear parameters estimation designed in this paper is 1.1s. It presents a fast approximation property for the desired response. The rise time is 1.9s. The rapid response of the system is guaranteed.

When the sprayer passes the ground with an inclined angle, the spray boom system needs to track the angle of inclination. The desired tilt angle is selected to be 0.1 rad, and the tracking error curve is shown in Fig. 3. The settling time of the adaptive backstepping control algorithm for nonlinear parameter estimation designed in this paper is 0.55s. It also presents a fast approximation property for the desired response. The rise time is 0.3s. The rapid response of the system is guaranteed.

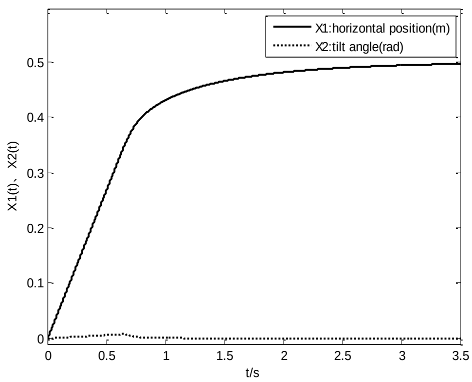


Figure 2. Expected horizontal position tracking curve

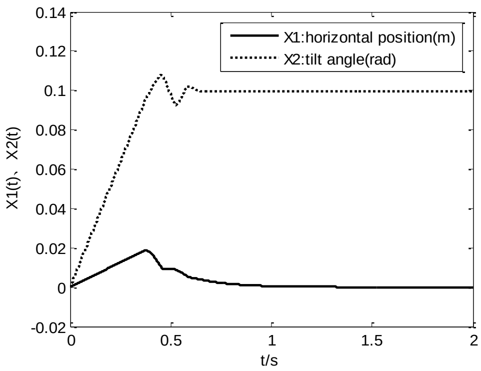


Figure 3. Expected tilt angle tracking curve

When the parameters of the boom control system drift due to changes in environmental conditions, the adaptive backstepping control algorithm is verified by simulation to suppress the drift of uncertain parameters. The uncertain parameters of the boom system are changed in a small random range, and the simulation test of the expected vertical displacement of 0.1 m is performed. The simulation is shown in Fig. 4. It can be seen from the figure that the settling time of the algorithm designed in this paper is 0.6s and the rise time is 0.4s. For comparison, the tracking test is also performed on the backstepping control algorithm without adaptive law under the same conditions. The simulation is shown in Fig. 5. The controller without adaptive law does not significantly suppress the parameter drift, indicating that the adaptive backstepping algorithm has a good compensation ability for parameter drift.

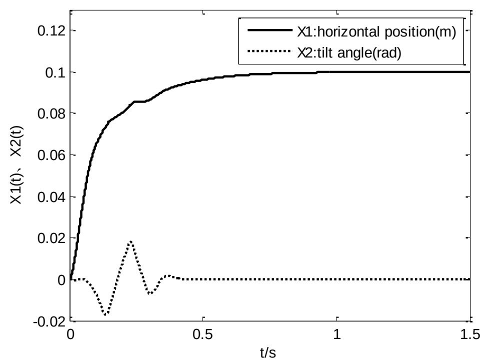


Figure 4. Tracking curve of the controller

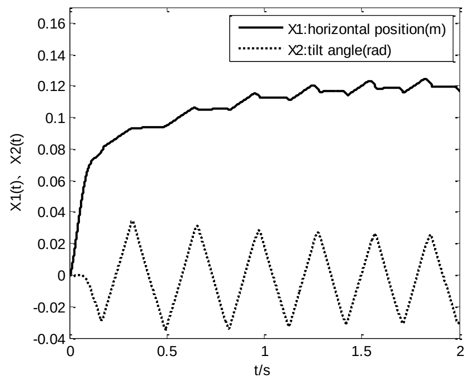


Figure 5. Tracking curve without adaptive law

## VI. Conclusions

Aiming at spray boom vibration and tilt ,which are caused by boom sprayer during its walking, and hydraulic cylinder parameter drift due to environmental changes, a two-asymmetric hydraulic cylinder electro-hydraulic servo system model has been established for the spray boom. Then an adaptive backstepping control method with improved Lyapunov function has been designed. It not only directly targets the nonlinear factors of the hydraulic cylinders in the system, but also effectively suppresses the drift of uncertain parameters in the system. It shows that the control method has strong robustness, moreover, has been expand a single hydraulic cylinder to a dual hydraulic cylinder with coupling. Theoretically, broadening the application range of asymmetric hydraulic cylinders has certain reference value.

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